Honors Algebra 2B Exam Review

The semester B examination for Honors Algebra 2 will consist of two parts. Part 1 will be selected response on which a calculator will not be allowed. Part 2 will be short answer on which a calculator will be allowed.

The following symbol applies to this review:

Indicates that a student should be prepared to complete a question like this with or without a calculator.

- If a calculator is used to find points on a graph, the appropriate calculator function (i.e. zero, intersect, minimum or maximum) should be used. The trace function should not be used.
- Decimal approximations must be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of any function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

The formulas below will be provided in the examination booklet.

Compound Interest:

Continuously: $A = Pe^{rt}$ $\qquad$ n times per year: $A = P\left(1+\frac{r}{n}\right)^{nt}$

Arithmetic Sequence and Series:

$$a_n = a_1 + (n-1)d \qquad S_n = n\left(\frac{a_1 + a_n}{2}\right) = n\left(\frac{2a_1 + (n-1)d}{2}\right)$$

Geometric Sequence and Series:

$$a_n = a_1 r^{n-1} \qquad S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

Conic Sections:

Circle	$x^2 + y^2 = r^2$ Center $(0,0)$ radius r
Parabola	$x^2 = 4py$ or $y = \frac{1}{4p}x^2$ opens up if $p > 0$, opens down if $p < 0$ Vertex $(0,0)$ $y^2 = 4px$ or $x = \frac{1}{4p}y^2$ opens right if $p > 0$, opens left if $p < 0$ Vertex $(0,0)$ $p =$ distance from vertex to focus and from vertex to directrix
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ major axis horizontal Center $(0,0)$ $a > b$ $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ major axis vertical Center $(0,0)$ $a > b$ $a =$ distance from center to vertex $c =$ distance from center to focus $c^2 = a^2 - b^2$
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ transverse axis horizontal Center $(0,0)$ $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ transverse axis vertical Center $(0,0)$ $a =$ distance from center to vertex $c =$ distance from center to focus $c^2 = a^2 + b^2$

Circle	$(x-h)^2 + (y-k)^2 = r^2$ Center (h,k) radius r
Parabola	$(x-h)^2 = 4p(y-k)$ or $y - k = \frac{1}{4p}(x-h)^2$ opens up if $p > 0$, opens down if $p < 0$ Vertex (h,k) $(y-k)^2 = 4p(x-h)$ or $x - h = \frac{1}{4p}(y-k)^2$ opens right if $p > 0$, opens left if $p < 0$ Vertex (h,k) $p =$ distance from vertex to focus and from vertex to directrix
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ major axis horizontal Center (h,k) $a > b$ $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ major axis vertical Center (h,k) $a > b$ $a =$ distance from center to vertex $c =$ distance from center to focus $c^2 = a^2 - b^2$
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ transverse axis horizontal Center (h,k) $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ transverse axis vertical Center (h,k) $a =$ distance from center to vertex $c =$ distance from center to focus $c^2 = a^2 + b^2$

UNIT 4: Sequences and Series

1. Given the sequence 3, 12, 48, 192, …

   a. Write the recursive rule for the sequence.

   $a_1 =$ _______

   $a_n =$ _______________

   b. Write an explicit rule for the sequence.

   $a_n =$ _______________

   c. What is the 15^{th} term?

   d. What is the sum of the first fifteen terms of the sequence?

2. Given the sequence 1, 4, 7, 10, …

   a. Write the recursive rule for the sequence.

   $a_1 =$ _______

   $a_n =$ _______________

   b. Write an explicit rule for the sequence.

   $a_n =$ _______________

   c. What is the 60^{th} term?

   d. What is the sum of the first sixty terms of the sequence?

3. A balloon filled with 5000 cm^3 of helium loses one fourth of its helium per day. How much helium will it have lost by the end of the ninth day?

4. A theater is designed with 50 seats in the first row, 53 seats in the second row, 56 seats in the third row, and so on. If there are 30 rows in the theater, how many seats are there?

UNIT 5 Power and Radical Functions

5. Rewrite using rational exponent notation.

a. $\sqrt[3]{17}$ b. $\sqrt[4]{x^3}$

6. Rewrite using radical notation.

a. $(-7)^{\frac{4}{3}}$ b. $x^{\frac{2}{5}}$

7. Evaluate.

a. $9^{\frac{1}{2}} \cdot 81^{\frac{3}{4}}$ b. $27^{-\frac{2}{3}}$

c. $\left(\frac{3^4}{7^4}\right)^{-\frac{1}{4}}$ d. $\left(2^{\frac{1}{5}} \cdot 2^{\frac{1}{2}}\right)^{10}$

8. Simplify each of the following. Assume all variables are positive.

a. $\left(a^{\frac{1}{3}}b^{\frac{3}{4}}\right)^{12}$ b. $4x^{-\frac{2}{3}} \cdot 6x^{-\frac{1}{6}}$ c. $\dfrac{x^{\frac{3}{5}}}{x^{\frac{2}{3}}}$

d. $\dfrac{a^{\frac{2}{3}}bc^{-\frac{3}{4}}}{a^{-\frac{1}{3}}b^{-2}c^{\frac{7}{4}}}$ e. $\left(x^5\right)^{\frac{3}{10}}$ f. $\sqrt[4]{81x^4}$

g. $\sqrt[3]{8x^{18}}$ h. $\left(\sqrt[3]{x^5} \cdot \sqrt[3]{x^4}\right)^{-4}$ i. $\dfrac{\sqrt[5]{a^{10}}}{\sqrt[5]{32a^2} \cdot \sqrt[5]{a^{13}}}$

9. Graph each function and state its domain and range.

a. $y = \sqrt{x-3} + 5$ b. $y = \sqrt[3]{x+5}$

10. Describe how to obtain the graph of $g(x)$ from the graph of $f(x)$.

a. $g(x) = -\sqrt{x-5},\ f(x) = \sqrt{x}$

b. $g(x) = \sqrt[3]{x+8} - 2,\ f(x) = \sqrt[3]{x}$

c. $g(x) = \sqrt{-x+2} + 7,\quad f(x) = \sqrt{x}$

11. The approximate time *t*, in seconds, that it takes an object to fall a distance *d*, in feet, is $t = 0.92\sqrt{\frac{d}{16}}$.

Sammi is parachuting and falls 1600 feet before she opens her parachute. How long does it take Sammi to fall this distance?

12. A car traveling on Interstate 270 was involved in an accident. To approximate the speed of the car at the time of the accident, the police use the function $s = \sqrt{30(0.6)d}$, where *s* is the speed in miles per hour and *d* is the length of the skid mark in feet.
The driver said he was traveling at 40 miles per hour.

What should be the length of the skid mark?

13. Solve. Check for extraneous roots.

a. $4x^{\frac{3}{4}} = 32$ b. $6x^{\frac{2}{5}} - 20 = 4$ c. $\sqrt{5x-7} + 6 = 4$

d. $\sqrt[3]{x-18} = -6$ e. $\sqrt{x+9} = x - 11$

14. Match each function to its graph:

a. $f(x) = \frac{1}{4}x^3 + 3$ b. $f(x) = \sqrt{x} + 2$ c. $f(x) = \sqrt{x+2}$

d. $f(x) = \sqrt[3]{x-1} + 2$ e. $f(x) = \sqrt[3]{x+2} - 1$

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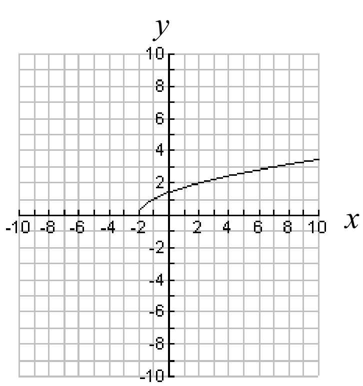

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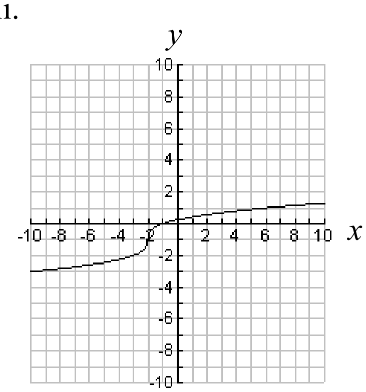

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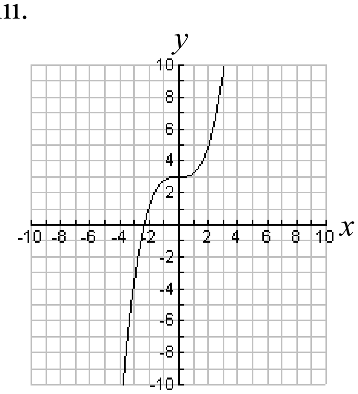

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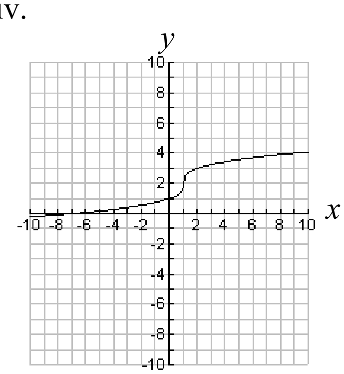

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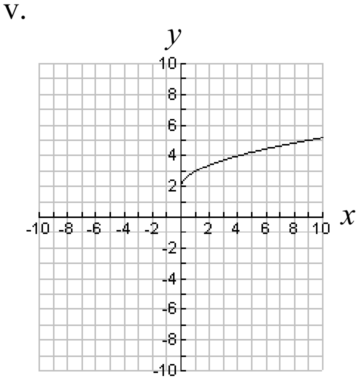


UNIT 6 – Exponential and Logarithmic Functions

15. Graph the function. Identify the *y*-intercept, the asymptote, state the domain and range, and whether the function is increasing or decreasing.

a. $y = 4^{-x}$ b. $y = 4^{x} + 3$ c. $y = \left(\frac{2}{3}\right)^{x}$

16. In 1900, the population of a town was 5000. The population increased at an average rate of 2.5% per year.

- What was the population in 1920?
- What was the population in 1950?

17. An account pays 4% interest compounded monthly. You deposit $6000 into the account. If you do not deposit or withdraw money from the account, how much will be in the account in 5 years?

18. A luxury car originally costs $75,000. It depreciates at a rate of 25% per year.

- What will the car be worth after 5 years?
- What will the car be worth after *t* years?

19. A town has a population of 250,000. Due to a loss of jobs in the area, the population of the town decreases at a rate of 5% per year.

- Write an exponential model that describes this situation.
- What is the expected population of the town in 10 years?

20. You deposit \$1000 in an account to save money for a car. The bank pays 7.4% interest. If you neither deposit nor withdraw any money from the account, how much will be in the account after 6 years if the interest is compounded continuously?

21. Graph the function. State the y-intercept, the asymptote, the domain, the range, and whether the function is increasing or decreasing.

a. $y = e^{-2x}$ b. $y = 3e^{x-1} + 2$

22. In a bacterial culture, the number b of bacteria present is modeled by $b = 15,000e^{0.3t}$, where t represents time in hours since 12:00 noon.

a. How many bacteria will be present at 5:00 PM?

b. How long will it take for there to be over 120,000 bacteria?

23. The population of a county today is 2000 and is growing exponentially according to the function $\boldsymbol{A(t) = Pe^{rt}}$. Ten years from today, the population is expected to be 7000. At what rate is the county growing?

24. Rewrite each equation in exponential form.

a. $\log_{10} \frac{1}{10000} = -4$ b. $\log_6 216 = x$

c. $\log 5 = x$ d. $\ln 7 = m$

25. Rewrite each equation in logarithmic form.

a. $32^{\frac{4}{5}} = 16$ b. $e^{3x} = 2$ c. $10^5 = 100{,}000$

26. Evaluate each expression.

a. $\log_3 81$ b. $\log_{23} 1$ c. $\log 100$

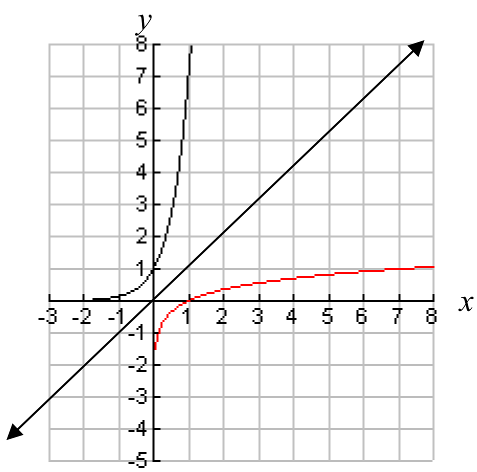
d. $\log_9 27$ e. $\log_8 16$

27. Simplify each expression using the inverse relationship between logarithms and exponents.

a. $11^{\log_{11} x}$ b. $\log_3 3^x$ c. $\ln e^x$

d. $e^{\ln x}$ e. $\log_5 125^x$ f. $\log 10{,}000^x$

28. The graph below illustrates a relationship between the functions $y = 7^x$ and $y = \log_7 x$.



- What is the relationship between the two functions?

- Explain how the relationship is determined from the graph.

29. Graph the function. State the asymptote, the domain, the range, and whether the function is increasing or decreasing.

a. $y = \log_{10} x - 2$

b. $y = \ln(x-2)$

30. Solve each equation.

a. $4^x = 2^5$

b. $2 \cdot 5^{\frac{x}{4}} = 250$

c. $10^x = 40$

d. $e^x = 9$

e. $5^{2x} = 625$

f. $\ln x = 3$

g. $\log_2 8 = x$

h. $3\log_2 x = 15$

i. $\log_x 64 = 2$

j. $\log_4(x+9) = 2$

k. $10^{(2x-3)} + 5 = 26$

l. $20\left(\frac{1}{2}\right)^{\frac{x}{3}} = 5$

m. $4^{(x-3)} = 8^{2x+1}$

n. $5\ln(x+4) - 17 = 8$

31. Write an exponential function of the form $y = ab^x$ whose graph passes through the given points.

a. $(0,5)$ *and* $(1,15)$

b. $(1,20)$ *and* $(3,500)$

32. For each set of data below:
    - Based on the patterns of change, which type of function best fits the data: linear, quadratic, radical, exponential, or logarithmic?
    - Write the function that best fits the data.

x	1	2	3	4	5
$f(x)$	4	16	64	256	1024

x	1	2	3	4	5
$f(x)$	15	20	25	30	35

x	2	4	8	16	32
$f(x)$	1	2	3	4	5

x	0	1	2	3	4
$f(x)$	0	1	4	9	16

x	0	1	2	3	4
$f(x)$	400	200	100	50	25

x	0	4	9	16	25
$f(x)$	0	2	3	4	5

33. The number of bacteria (y) in a culture is a function of the number of hours (x) the culture has been growing. The initial number of bacteria present in a culture is 50; the number of bacteria in the culture for each hour is represented in the table below.

x	0	1	2	3	4
y	50	150	450	1350	4050

- What type of function best models this data?
- Write a function that models the data.
- Predict how many bacteria there will be after 9 hours.

34. Match each equation to its graph:

a. $y = 2 \cdot 3^x$ b. $y = 4^{x+2}$

c. $y = 4^x + 2$ d. $y = \left(\frac{1}{3}\right)^{x-2}$

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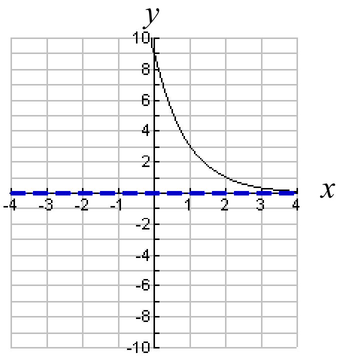

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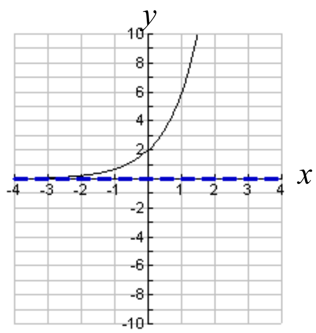

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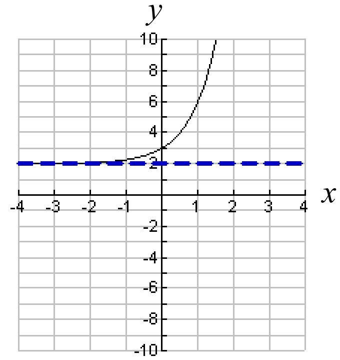

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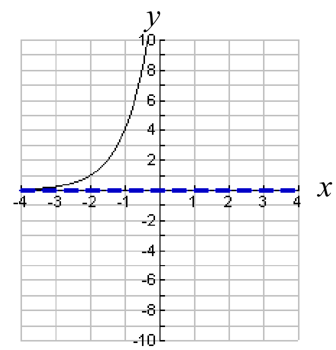
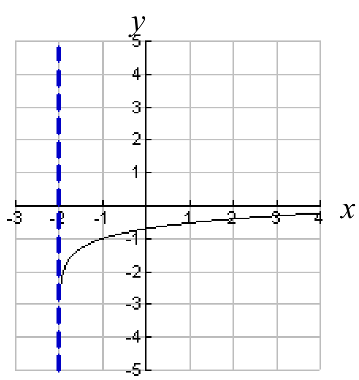


35. Match each equation to its graph:

a. $y = 2e^{x-3} - 3$ b. $y = e^{-x}$ c. $y = e^x$

d. $y = \log_{10} x$ e. $y = \log_{10}(x+2) - 1$

i.



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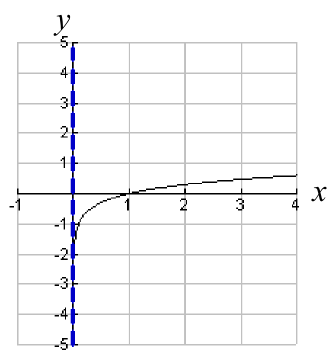

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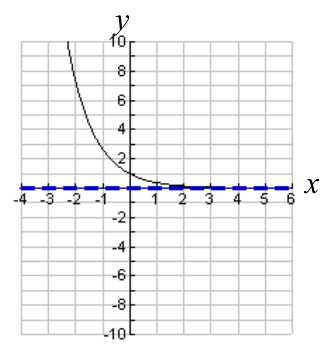

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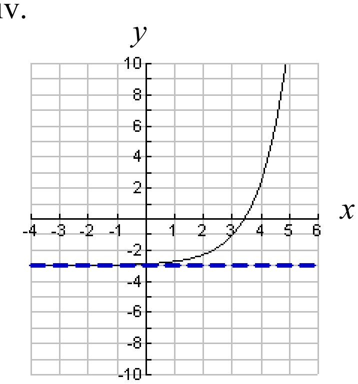

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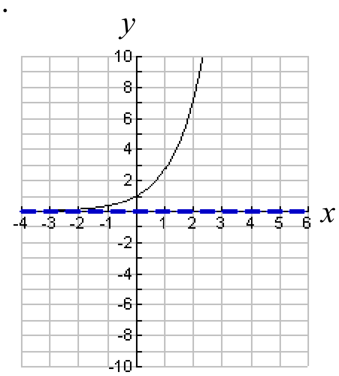


UNIT 7 – Rational Functions

36. Write an equation relating the variables. Use k for the constant of variation.

    a. s varies directly as r.

    b. y varies inversely as x.

    c. t varies directly as r and inversely as s.

    d. V varies jointly with r and h.

    e. F varies jointly with m and n, and inversely as the square of v.

37. If y varies inversely as x, and $y = -2$ when $x = 25$, find x when $y = -40$.

38. The time t required to empty a pool varies inversely as the rate r of the pump. One pool takes 60 minutes to empty at the rate of 230 gallons per minute.
    - Write an equation that represents the relationship between t and r.
    - Find the constant of variation k.
    - How long will it take to empty the same pool with a pump that can empty it at the rate of 300 gallons per minute?

39. If y varies directly with x and inversely as z , and $y = 9$ when $x = 2$ and $z = 12$, find y when $x = 3$ and $z = 10$.

40. The intensity of light (I) measured in lux is inversely proportional to the square of the distance (d) between the light source and the object illuminated. The intensity of light from an overhead fixture to a dining room table 5 feet below the fixture is 23 lux.
    - Write an equation that represents the relationship between I and d.
    - What will the intensity be if the fixture is raised to a height of 6.5 feet above the table?

41. Graph each function. Identify the horizontal and vertical asymptotes. State the domain and range.

a. $y = \dfrac{2}{x-4}$

b. $y = \dfrac{x+4}{x-3}$

c. $y = \dfrac{2}{x+1} + 4$

d. $y = \dfrac{4x-2}{x+3}$

42. Perform the indicated operations. Simplify the result.

a. $\dfrac{x^2-4}{x^2-3x-10} \cdot \dfrac{x^2-25}{x-2}$

b. $\dfrac{6}{x^2-2x-15} \div \dfrac{12}{x-5}$

c. $\dfrac{3x^2-48}{x^2+7x+12} \div \dfrac{x^2-2x-8}{x^2+5x+6}$

43. Perform the indicated operations and simplify.

a. $\dfrac{x}{x+5} + \dfrac{5}{x+4}$

b. $\dfrac{4x}{2x+1} - \dfrac{2x}{x-3}$

c. $\dfrac{\frac{x}{3}-4}{5+\frac{1}{x}}$

d. $\dfrac{2x+3}{x^2+4x+4} + \dfrac{5}{x+2}$

e. $\dfrac{7+\frac{5}{x+3}}{\frac{9}{x+3}+\frac{8}{x-4}}$

44. Solve each equation. Check for extraneous roots.

a. $\dfrac{-2}{x-1} = \dfrac{-8}{x+1}$

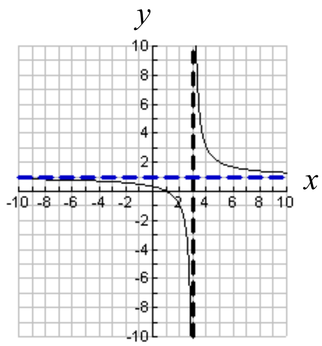
b. $\dfrac{x}{2x+7} = \dfrac{x-5}{x-1}$

c. $x + \dfrac{x}{x-2} = \dfrac{2}{x-2}$

45. Match each equation to its graph.

a. $y=\dfrac{2}{x-3}$ b. $y=\dfrac{2}{x-3}+1$ c. $y=\dfrac{x-3}{x+2}$ d. $y=\dfrac{3x+4}{x-2}$

i.



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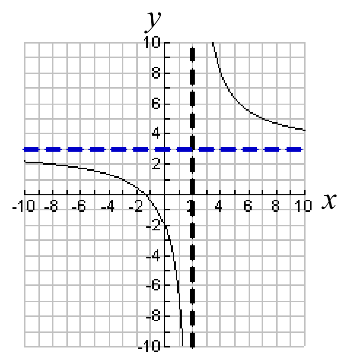

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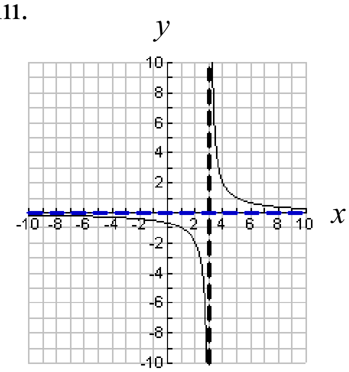

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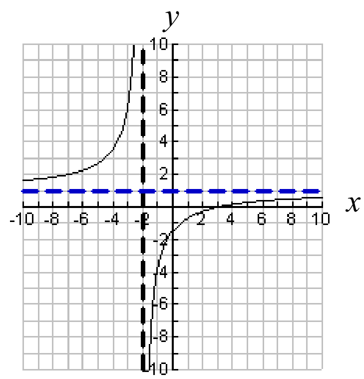

UNIT 8 – Conic Sections

46. Identify each locus of points in the plane.

a. The set of all points such that the sum of the distances from two fixed points to each point on the locus is a constant.

b. The set of all points such that the difference of the distances from two fixed points to each point on the locus is a constant.

c. The set of all points equidistant from a given point.

d. The set of all points that are equidistant from a given point and a given line.

47. Sketch the graph of each equation. Identify the focus and directrix.

a. $4y^2 = x$ b. $x^2 + 36y = 0$

48. Write the equation of a circle that passes through the point $(-3, 4)$ and whose center is the origin.

49. Sketch the graph of the equation $x^2 + y^2 = 16$ and state the center and radius.

50. An ellipse has a center at $(0,0)$. One vertex is at the point $(0,10)$ and a co-vertex is at the point $(-8,0)$. Write an equation of the ellipse.

51. Describe the graph of the equation $\frac{x^2}{36} + \frac{y^2}{16} = 1$.

52. Sketch the graph of each equation. Identify the center and the vertices.

a. $\frac{x^2}{64} - \frac{y^2}{36} = 1$ b. $9y^2 - 4x^2 = 36$

53. Match each equation to its graph.

a. $y^2 = -8x$ b. $x^2 + y^2 = 49$ c. $\frac{x^2}{9} + \frac{y^2}{36} = 1$

d. $\frac{x^2}{25} + \frac{y^2}{4} = 1$ e. $\frac{y^2}{16} - \frac{x^2}{4} = 1$ f. $\frac{x^2}{1} - \frac{y^2}{25} = 1$

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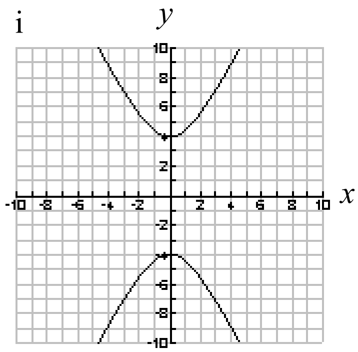

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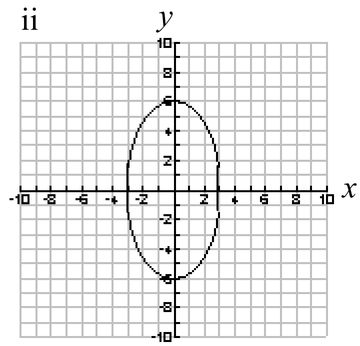

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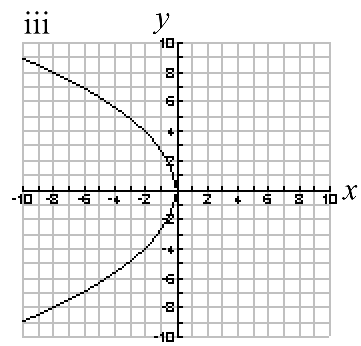

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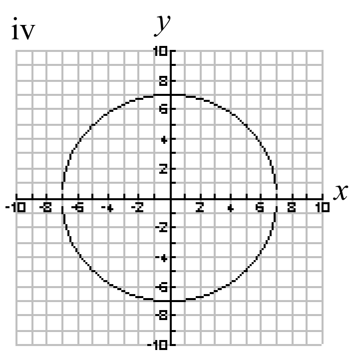

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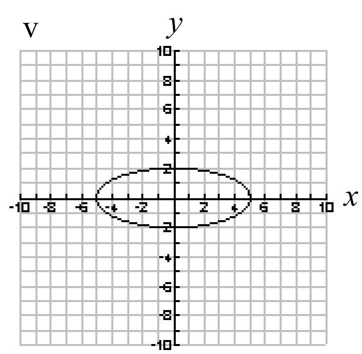

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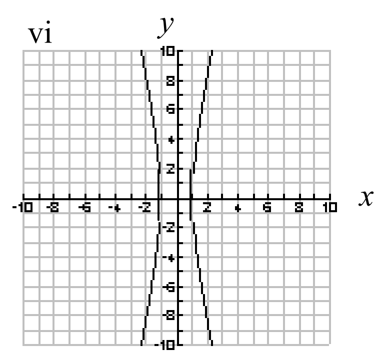


54. For each of the following equations, name the conic section.
- For parabolas, name the focus, directrix, equation of the axis of symmetry, and the direction in which the parabola opens.
- For circles, name the center and the length of the radius.
- For ellipses, name the vertices, co-vertices, and foci.
- For hyperbolas, name the foci and the vertices.

a. $\frac{1}{2}x = y^2$

b. $\frac{x^2}{25} - \frac{y^2}{64} = 1$

c. $x^2 + y^2 - 49 = 0$

d. $\frac{x^2}{121} + \frac{y^2}{49} = 1$

55. For each of the following equations of conic sections:
- Identify the type of conic.
- Write the equation in standard form.

a. $x^2 + 4y - 12 = 0$

b. $x^2 + 9y^2 + 2x - 18y = -1$

c. $x^2 + y^2 - 4x + 8y - 5 = 0$

d. $9x^2 - 4y^2 - 36x + 8y - 4 = 0$

56. Match each equation to its graph.

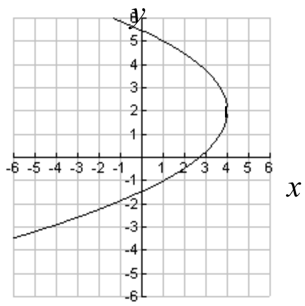
a. $(x-3)^2+(y+2)^2=4$

b. $(y-2)^2=-3(x-4)$

c. $\dfrac{x^2}{4}-(y-2)^2=1$

d. $\dfrac{(x-3)^2}{16}+\dfrac{(y+3)^2}{9}=1$

i.



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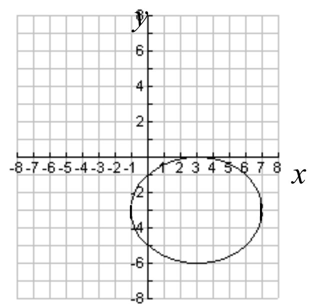

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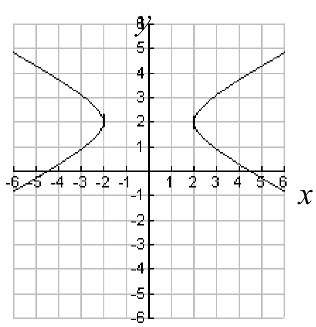

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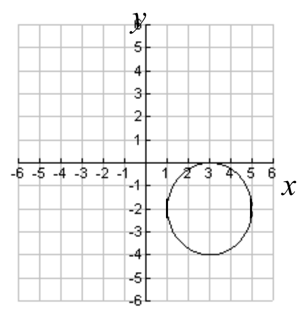


57. Solve each system of equations for *x* and *y*.

a. $x=4y-3$
$x^2-4y^2=9$

b. $5x^2+y^2=30$
$9x^2-y^2=-16$

Practice Student Produced Response questions

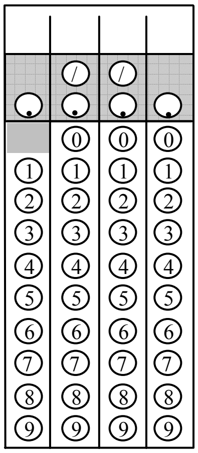
58. Evaluate $25^{-\frac{3}{2}}$

59. Evaluate $\sqrt[3]{16} \div \sqrt[3]{2}$

60. Solve $x^{\frac{5}{3}} = 32$

61. Evaluate $\log 10^5$

58



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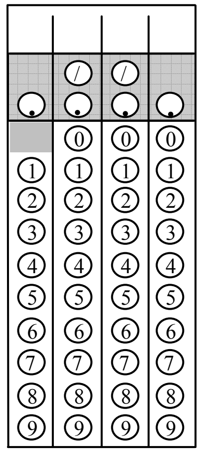

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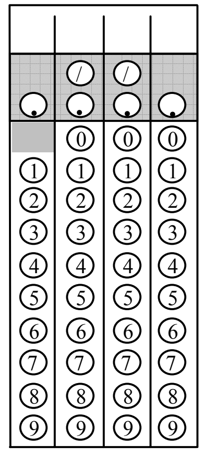

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