Montgomery County Public Schools
High School Curriculum Framework
Mathematics
Algebra 2/Algebra 2 with Analysis
and
Precalculus/Precalculus with Analysis

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Table of Contents

High School Curriculum Framework for Mathematics

Introduction

The High School Curriculum Framework for Mathematics contains the same goals and overarching enduring understandings as those in the preK-8 Curriculum Framework for Mathematics. The descriptions of the content and instructional approach are modifications of those in the preK-8 Curriculum Framework for Mathematics. These modifications reflect the content and instruction in High School Mathematics. The framework contains the Scope and Sequence of indicators for Algebra 2/Algebra 2 with Analysis and Precalculus/Precalculus with Analysis. The indicators in bold italic type are for the courses with analysis.

The Montgomery County Public Schools mathematics program includes courses beyond the HSA courses that lead to Advanced Placement Mathematics. All of the High School Mathematics courses are shown in the flow chart of High School Mathematics Sequences.

Goal:
The goal of the Montgomery County Public Schools pre-K-12 mathematics program is for all students to achieve mathematical proficiency by developing both conceptual understanding and procedural fluency. The end result is the ability to think and reason mathematically and use mathematics to solve problems in authentic contexts.

Overarching Enduring Understandings:
- Mathematics is the study of patterns and relationships.
- Mathematics is a language consisting of carefully defined terms and symbols.
- Mathematics is a tool used to solve problems in everyday life.
- Technology influences the mathematics that is taught and essential for our world.

The Content of Mathematics:
Mathematics is a tool we use to understand and interpret our world. In our increasingly technological economy, those who can understand and apply mathematics have significantly enhanced opportunities to achieve success in continuing education and in life. The key to opening the door to these opportunities is a deep understanding of important mathematical concepts and procedures.

The integration of both mathematical concepts and processes is essential for meaningful understanding of mathematics. In this framework, the concepts of mathematics are organized under the strands: Algebra/Patterns/Functions, Geometry, Measurement, Statistics, Probability, and Number Relationships and Computation. These concepts are developed through the four mathematical processes that are organized into four strands: Problem Solving, Communication (including representation), Reasoning, and Connections.

The mathematical content must be coherent and vertically articulated across the grades.
- In the elementary school years, students develop proficiency with number concepts and operations. For this to occur, students' experiences with the concept of number must be connected to mathematical concepts in geometry, algebraic reasoning, and data analysis. At the same time, proficiency with mathematical facts and skills must be developed so that students are facile in their application of mathematics to solve problems.
- In the middle grades, students extend their mathematical proficiency through their work with rational numbers, proportional reasoning, measurement, and data analysis. They continue the development of a deep understanding of important algebraic and geometric concepts as well as mathematical ways of thinking. The expectation is for all students to be successful in the formal study of algebra and other academically challenging mathematics courses. The PreK-7 indicators and the checked grade 8 indicators identify the prerequisite knowledge and understanding necessary for success in Algebra I, Geometry, and beyond.
- In high school, all students pursue rigorous mathematics coursework that includes algebra, functions, geometry, statistics, calculus, and discrete mathematics. Students must learn important mathematical concepts with understanding so that they are prepared for a wide range of career and educational choices. (Principles and Standards for School Mathematics, 2000) Multiple representations enable students to make connections and analyze situations both within mathematics and in other disciplines. Problem solving provides the context for the development of formal definitions and procedures. As students reflect upon and clarify their thinking, their knowledge, communication, and appreciation of mathematics will deepen.

Instructional Approach:
Learning with understanding is essential for developing mathematical proficiency. According to the National Research Council's report *Adding It Up*, mathematical proficiency implies expertise in handling mathematical ideas. Students with mathematical proficiency understand basic concepts, are fluent in performing basic operations, reason clearly, formulate, represent, and solve mathematical problems, and maintain a positive outlook toward mathematics. (Kilpatrick, 2001) These components of mathematical proficiency are interwoven and interdependent. Instruction in high school mathematics must help students refine and extend previously learned strategies for working with numbers and operations to more complex operations and concepts. The interdependence and connections among all six mathematical strands fosters the development of mathematical proficiency. Students must be actively engaged in learning experiences that are designed to deepen, connect, and build on students' knowledge. Communication is an essential part of mathematics education. Mathematics has many words and symbols that are unique to the discipline. Instruction must provide students with opportunities for speaking, reading, writing, representing, and listening in mathematics classrooms so that they will learn to communicate mathematically. (Principles and Standards for School Mathematics, 2000) Technology is a tool for investigation and problem solving that enhances learning of mathematics and actively engages students. The use of technology should support the development of mathematical proficiency.

Mathematics teaching and learning must be challenging and rigorous with an emphasis on reasoning and problem solving. The development of logical reasoning is an essential component in learning mathematics. The logical reasoning inherent in the study of mathematics allows for applications to a variety of situations in which solutions to problems can be found with accuracy. (California Math Standards) The curriculum makes a distinction between problem solving as a general process and the solution of specific word problems that demonstrate application of mathematical skills. A mathematical problem is something that students do not already know how to do. Problem solving is the process of transforming something that the students do not know how to do into something familiar. (Steen, 1997) The mathematical problem solving situations that students encounter should include problems that require broader thinking than traditional word problems demand. (Burns, 1992) Word problems are a means for practicing computations and procedures. For example, a traditional word problem might ask: *A father is 20 years older than twice his son's age. If the father is 50 years old, how old is the son?* The intent of this problem is to practice solving a one variable equation. Problem solving, on the other hand, should require students to develop a plan, execute the plan, and establish a purpose for learning a mathematical concept. For example, the previously cited word problem becomes a problem solving situation when it is restated as follows. *Chris charges $20 plus $2 per page to type a term paper. Tom charges $10 plus $3 per page to type a term paper. Which person will charge the least amount of money to type a term paper? Use mathematics to justify your answer.* The problem situations that students encounter should extend their range of problem solving strategies, deepen their mathematical understanding, and provide a meaningful context for learning mathematics. Problem solving must occur at every grade level and be the primary focus in mathematics instruction.

Differentiated instruction addresses student strengths, interests, and learning styles and should be paced to make the curriculum accessible to everyone. Flexible and varied grouping practices enhance the opportunity to receive expanded, intensive, enriched, and accelerated curriculum at all instructional levels as warranted by students' needs. A balance needs to be achieved so that all students have the opportunity to work in homogenous and heterogeneous groups. The curriculum is designed so that all students have the necessary skills and understanding for

success in further study of mathematics or a career. The pathway for students to take Calculus in high school or college is clearly laid out in the high school sequence of courses. Differentiation and grouping practices must be implemented to ensure that all students in both honors and regular courses are prepared for the next level of study in mathematics.

Assessment is an ongoing process that guides instruction and monitors student progress to include mastery of mathematics content and higher level thinking skills. Pre-assessment, formative, and summative assessments provide for student, peer, and teacher evaluation. These types of assessment enable teachers to modify their instruction to support improved learning at each grade level for all students. Assessment should be focused on the development and achievement of mathematical proficiency. (Kilpatrick, 2001)

Documents and Concepts Considered in this framework:

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California Math Standards. < http://www.step.k12.ca.us/resources/MathStandards.html>.

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High School Core Learning Goals: Mathematics; Maryland School Performance Program. Baltimore, MD: Maryland State Department of Education, 1999. 5 June 2001 <http://www.mdk12.org/mspp/high_school/what_will/mathematics/index.html> or <http://www.mdk12.org/mspp/high_school/what_will/mathematics/mathematics_goals99.pdf>

Keys to Math Success A Report from the Maryland Mathematics Commission. Baltimore, MD: Maryland State Department of Education, June 2001.

Kilpatrick, Jeremy, Jane Swafford, and Bradford Findell, eds. Adding It Up: Helping Children Learn Mathematics. Washington, DC: National Academy Press, 2001.

Mullis, Ina V.S. et al. Mathematics Benchmarking Report TIMSS 1999 – Eighth Grade. Boston, MA: International Study Center, Boston College, International Association for the Evaluation of Educational Achievement. April 2001 <http://www.timss.org>

Maryland Mathematics Content Standards. Baltimore, MD: Maryland State Department of Education, 2000. 5 June 2001 <http://www.mdk12.org/practices/support_success/mspap/mathematics/content_standards.html> or <http://www.mdk12.org/share/standards/constds_math.pdf>

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Marzano, Robert J., and Debra J. Pickering. Dimensions of Learning. 2nd ed. Alexandria, VA: Association for Supervision and Curriculum Development; Aurora, CO: Mid-continent Regional Educational Laboratory, 1997.

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Primary Mathematics Syllabus. Singapore: Ministry of Education, Curriculum Planning and Development Division, 2000.

Principles and Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics, 2000.

Scholastic Achievement Test. The College Board/Educational Testing Service. <http://www.ets.org/satets.html>.

Science for All Americans: A Project 2061 Report on Literacy Goals in Science, Mathematics, and Technology. Washington, DC: American Association for the Advancement of Science, 1989

Steen, Lynn Arthur. Why Numbers Count Quantitative Literacy for Tomorrow's America. New York: College Entrance Examination Board, 1997

Tomlinson, Carol Ann. The Differentiated Classroom: Responding to the Needs of All Learners. Alexandria, VA: Association for Supervision and Curriculum Development, 1999.

Wiggins, Grant, and Jay McTighe. Understanding by Design. Alexandria, VA: Association for Supervision and Curriculum Development, 1998.

Content Standard 1.0 Knowledge of Patterns, Algebra and Functions

Students will algebraically represent, model, analyze, and solve mathematical and real-world problems involving functional patterns and relationships.

Expectation 1.1

The student will represent functions and relations numerically, graphically, and algebraically.

Indicators for Algebra 2 and ***Algebra 2 with Analysis***		Indicators for Precalculus and ***Precalculus with Analysis***	
1.1.A2.1	write a polynomial function given its real zeros or a graph with real zeros.	1.1.PC.1	write a rational function or expression in an equivalent form.
1.1.A2.2	***write a polynomial function given its real or complex zeros.***	***1.1.PC.2***	***write a rational function or expression in an equivalent form, including partial fractions.***
1.1.A2.3	write a radical function or expression as an equivalent power function or expression.	1.1.PC.3	write an exponential function or expression in an equivalent form using laws of exponents
1.1.A2.4	write a rational function or expression in the form $\frac{1}{x^n}$ as an equivalent power function or expression.	***1.1.PC.4***	***write equivalent forms for rectangular and polar equations.***
1.1.A2.5	write the equation and describe the characteristics of a circle, ellipse, and hyperbola centered at the origin, and parabola with vertex at the origin given its graph.	1.1.PC.5	write equivalent equations for functions and relations in parametric and rectangular form.
1.1.A2.6	***write the equation and describe the characteristics of a conic section given its graph.***	1.1.PC.6	write a vector equation of a line in 2-space.
1.1.A2.7	represent exponential functions, including base *e*, numerically, algebraically, and graphically.	***1.1.PC.7***	***write an equation of a line or plane in 3-space.***
1.1.A2.8	represent logarithmic functions, including base *e*, numerically, algebraically, and graphically.	1.1.PC.8	represent a piece-wise function numerically, algebraically and graphically.
1.1.A2.9	represent radical functions numerically, algebraically and graphically.	1.1.PC.9	represent parametric functions and relations numerically, algebraically, and graphically.
1.1.A2.10	represent piece-wise functions involving linear, absolute value, and step functions numerically, algebraically, and graphically.	1.1.PC.10	represent a series using summation notation.
1.1.A2.11	represent a system of two or more linear equations in matrix form.	1.1.PC.11	graph rational functions and describe their properties.
1.1.A2.12	represent arithmetic and geometric sequences explicitly and recursively.	***1.1.PC.12***	***graph rational functions and describe their properties, including limit theory as it applies to determining their asymptotes and removable discontinuities.***
1.1.A2.13	represent circles, ellipses, and hyperbolas centered at the origin, and parabolas with vertex at the origin algebraically and graphically.	***1.1.PC.13***	***graph polar equations and describe their properties.***
1.1.A2.14	***represent conic sections algebraically and graphically.***	***1.1.PC.14***	***graph a point, line, or plane in 3-space.***
1.1.A2.15	graph rational functions with numerators and/or denominators that are linear polynomials and describe their properties.	1.1.PC.15	determine the period, amplitude, phase shift, and/or vertical shift of a trigonometric function represented graphically or algebraically.
1.1.A2.16	determine the sum and n^{th} term of an arithmetic or geometric series.	1.1.PC.16	determine the sum, if it exists, of an infinite geometric series.
		1.1.PC.17	apply the fundamental trigonometric identities.
		1.1.PC.18	expand and evaluate a series written in summation notation.

Content Standard 1.0 Knowledge of Patterns, Algebra and Functions

Students will algebraically represent, model, analyze, and solve mathematical and real-world problems involving functional patterns and relationships.

Expectation 1.2
The student will describe and apply properties of functions and relations.

Indicators for Algebra 2 and ***Algebra 2 with Analysis***		Indicators for Precalculus and ***Precalculus with Analysis***	
1.2.A2.1	describe functions using domain and range, one-to-one, increasing, decreasing, continuous, maximum and minimum values, and symmetry.	1.2.PC.1	describe the properties of rational functions, including domain, range, continuity, end behavior, horizontal and vertical asymptotes.
1.2.A2.2	describe and compare the characteristics of polynomial functions, given numerical, graphical, and algebraic representations including domain and range, increasing, decreasing, continuous, maximum and minimum values, end behaviors, symmetry, zeroes and their multiplicity, and turning points.	***1.2.PC.2***	***describe oblique asymptotes of rational functions.***
1.2.A2.3	describe the properties of exponential functions including domain and range, increasing, decreasing, continuous, maximum and minimum values, end behaviors, symmetry, asymptotes, and zeros.	1.2.PC.3	describe the properties of linear, quadratic, power, polynomial, rational, exponential, logarithmic, trigonometric, and inverse trigonometric functions
1.2.A2.4	describe the properties of logarithmic functions including domain and range, increasing, decreasing, continuous, maximum and minimum values, end behaviors, symmetry, asymptotes, and zeros.	1.2.PC.4	describe the properties of a piece-wise function.
1.2.A2.5	describe the inverse relationship between exponential and logarithmic functions numerically, graphically, and algebraically.	1.2.PC.5	describe the inverse relationship between trigonometric and inverse trigonometric functions, numerically, algebraically and graphically.
1.2.A2.6	describe the properties of radical functions.	1.2.PC.6	identify and distinguish between the graphs of linear, quadratic, power, polynomial, rational, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
1.2.A2.7	describe the properties of a piece-wise function involving linear, absolute value, and step functions.		
1.2.A2.8	describe the properties of rational functions with numerators and/or denominators that are linear polynomials, including domain, range, continuity, end behavior, horizontal asymptotes, and vertical asymptotes.		
1.2.A2.9	describe the properties of circles, ellipses, hyperbolas centered at the origin and parabolas with vertex at the origin.		
1.2.A2.10	***describe the properties of circles, ellipses, hyperbolas, and parabolas.***		
1.2.A2.11	apply finite differences to find the degree of polynomial functions.		

Content Standard 1.0 Knowledge of Patterns, Algebra and Functions

Students will algebraically represent, model, analyze, and solve mathematical and real-world problems involving functional patterns and relationships.

Expectation 1.3

The student will perform a variety of operations and geometrical transformations on functions and relations.

Indicators for Algebra 2 and ***Algebra 2 with Analysis***		Indicators for Precalculus and ***Precalculus with Analysis***	
1.3.A2.1	describe the effect of transformations on the graph of $f(x)$, including $af(x), f(x-h), \text{and } f(x)+k.$	1.3.PC.1	describe the effect of single or multiple transformations on the graph of $f(x)$, including $af(x), f(x-h), \ f(x)+k, f(ax), \lvert f(x)\rvert, \text{and } f(\lvert x\rvert).$
1.3.A2.2	describe the effect of transformations on graphs of exponential functions, $f(x)=a(b)^{x-h}+k.$	1.3.PC.2	describe the effect of transformations on graphs of exponential functions, $f(x)=a(b)^{cx}.$
1.3.A2.3	describe the effect of transformations on the graphs of radical functions, $f(x)=\sqrt[n]{(x-h)}+k.$	***1.3.PC.3***	***describe the effect of transformations on graphs of logarithmic functions.***
1.3.A2.4	perform operations on functions, including determining the composition of two functions.	1.3.PC.4	describe the effect of transformations on the graphs of trigonometric functions.
1.3.A2.5	***determine the domain of the composition of two functions.***	***1.3.PC.5***	***describe the effect of transformations on a function with a restricted domain.***
1.3.A2.6	determine whether two functions are inverses analytically and graphically.		
1.3.A2.7	determine the inverse of a function.		
1.3.A2.8	***determine the standard form for conics.***		
1.3.A2.9	***modify the domain of a function so that its inverse is a function.***		

Content Standard 1.0 Knowledge of Patterns, Algebra and Functions

Students will algebraically represent, model, analyze, and solve mathematical and real-world problems involving functional patterns and relationships.

Expectation 1.4

The student will use numerical, algebraic, and graphical representations of functions and relations in order to solve real world problems.

Indicators for Algebra 2 and ***Algebra 2 with Analysis***		Indicators for Precalculus and ***Precalculus with Analysis***	
1.4.A2.1	solve polynomial equations using graphs, the factor theorem, rational root theorem, and the quadratic formula.	1.4.PC.1	solve exponential equations, including base *e*, using various methods including laws of logarithms.
1.4.A2.2	solve exponential equations using graphs, the laws of exponents or the inverse relationship with logarithms.	1.4.PC.2	solve logarithmic equations, including base *e*, using laws of logarithms and exponents
1.4.A2.3	solve logarithmic equations using graphs and the inverse relationship with exponents.	1.4.PC.3	solve rational equations numerically, graphically, or algebraically.
1.4.A2.4	solve rational equations with linear denominators graphically, numerically, and algebraically.	***1.4.PC.4***	***solve rational inequalities using a numeric method.***
1.4.A2.5	solve radical equations graphically or algebraically, and check for extraneous roots.	1.4.PC.5	solve trigonometric equations.
1.4.A2.6	solve systems of two or more linear equations using a variety of methods.	***1.4.PC.6***	***solve systems of equations in polar form.***
1.4.A2.7	solve polynomial inequalities using the graph of the related polynomial function.	1.4.PC.7	interpret and solve problems involving exponential functions.
1.4.A2.8	solve polynomial inequalities of degree 2 algebraically**.**	1.4.PC.8	interpret and solve problems involving logarithmic functions.
1.4.A2.9	***solve polynomial inequalities of degree greater than 2 algebraically.***	1.4.PC.9	interpret and solve problems involving piece-wise functions.
1.4.A2.10	***solve quadratic systems of equations and inequalities.***	1.4.PC.10	interpret and solve problems involving trigonometric functions.
1.4.A2.11	interpret and solve problems involving polynomial functions.	1.4.PC.11	interpret and solve problems involving parametric functions and relations.
1.4.A2.12	interpret and solve problems involving exponential functions.	1.4.PC.12	make predictions using trigonometric or power mathematical models given a set of data.
1.4.A2.13	interpret and solve problems involving rational equations, including inverse and combined variation.		
1.4.A2.14	interpret and solve problems involving radical functions.		
1.4.A2.15	interpret and solve problems involving piece-wise functions including linear, absolute value, and step functions.		
1.4.A2.16	make predictions using quadratic, exponential, or logarithmic mathematical models given a set of data.		
1.4.A2.17	choose appropriate models, quadratic, exponential, or logarithmic, based on an analysis of the patterns of change in data.		
1.4.A2.18	apply the Fundamental Theorem of Algebra.		

Content Standard 2.0 Knowledge of Geometry and Measurement

Maryland Content Standard: Knowledge of Geometry
Students will apply the properties of one-, two-, and three-dimensional geometric figures to describe, reason, and solve problems about shape, size, position, and motion of objects.

Maryland Content Standard: Knowledge of Measurement
Students will identify attributes, units, and systems of measurements and apply a variety of techniques, formulas, tools, and technology for determining measurements.

Expectation 2.1
The student will describe relationships between geometric figures.

Indicators for Algebra 2 and ***Algebra 2 with Analysis***		Indicators for Precalculus and ***Precalculus with Analysis***	
2.1.A2.1	describe circles, ellipses, parabolas, and hyperbolas as a locus of points.	***2.1.PC.1***	***write equivalent rectangular and polar forms of points on the coordinate plane.***
		2.1.PC.2	represent a vector in 2-space by its magnitude and direction, its initial and terminal point, and its component form.
		2.1.PC.3	***represent a vector in 3-space by its magnitude and direction, its initial and terminal point, and its component form.***
		2.1.PC.4	describe and apply the relationship between the trigonometry of the right triangle and the unit circle.
		2.1.PC.5	describe and apply the relationship between the radian measure of a central angle of a circle and its intercepted arc.
		2.1.PC.6	***determine multiple polar form representations of a point.***
		2.1.PC.7	***identify the pole and the polar axis, and plot points given in polar form.***
		2.1.PC.8	define and graph the six circular functions.

Content Standard 2.0 Knowledge of Geometry and Measurement

Maryland Content Standard: Knowledge of Geometry
Students will apply the properties of one-, two-, and three-dimensional geometric figures to describe, reason, and solve problems about shape, size, position, and motion of objects.

Maryland Content Standard: Knowledge of Measurement
Students will identify attributes, units, and systems of measurements and apply a variety of techniques, formulas, tools, and technology for determining measurements.

Expectation 2.2
The student will identify units of measurement and apply a variety of techniques for determining measurement.

Indicators for Algebra 2 and ***Algebra 2 with Analysis***	Indicators for Precalculus and ***Precalculus with Analysis***	
	2.2.PC.1	write the value of an inverse trigonometric expression in radians.
	2.2.PC.2	determine the distance from a point to a line in 2-space.
	2.2.PC.3	***determine the distance from a point to a plane in 3-space.***
	2.2.PC.4	***determine the angular and linear velocities of an object moving at a constant speed on a circular path.***
	2.2.PC.5	evaluate a trigonometric expression using radian measure.
	2.2.PC.6	convert degree measure to radian measure.
	2.2.PC.7	measure indirectly using trigonometric relationships.

Content Standard 4.0 Knowledge of Number Relationships and Computation

Maryland Content Standard: Knowledge of Number Relationships and Computation
Students will describe, represent, and apply numbers and their relationships, and they will estimate and compute using mental strategies, paper/pencil, and technology.

Expectation 4.1
The student will describe and represent numbers and their relationships.

Indicators for Algebra 2 and ***Algebra 2 with Analysis***		Indicators for Precalculus and ***Precalculus with Analysis***	
4.1.A2.1	write equivalent forms for exponential and logarithmic expressions and equations.	***4.1.PC.1***	***write equivalent rectangular and polar forms for complex numbers.***
4.1.A2.2	write equivalent expressions involving radicals and exponents, including negative exponents.	***4.1.PC.2***	***represent complex numbers in polar form numerically and graphically.***
4.1.A2.3	represent complex numbers numerically and graphically.		
4.1.A2.4	***determine the magnitude of complex numbers.***		
4.1.A2.5	determine whether a square matrix has a multiplicative inverse.		
4.1.A2.6	identify numbers as real or complex, and distinguish among rational, irrational, imaginary, and complex numbers.		

Content Standard 4.0 Knowledge of Number Relationships and Computation

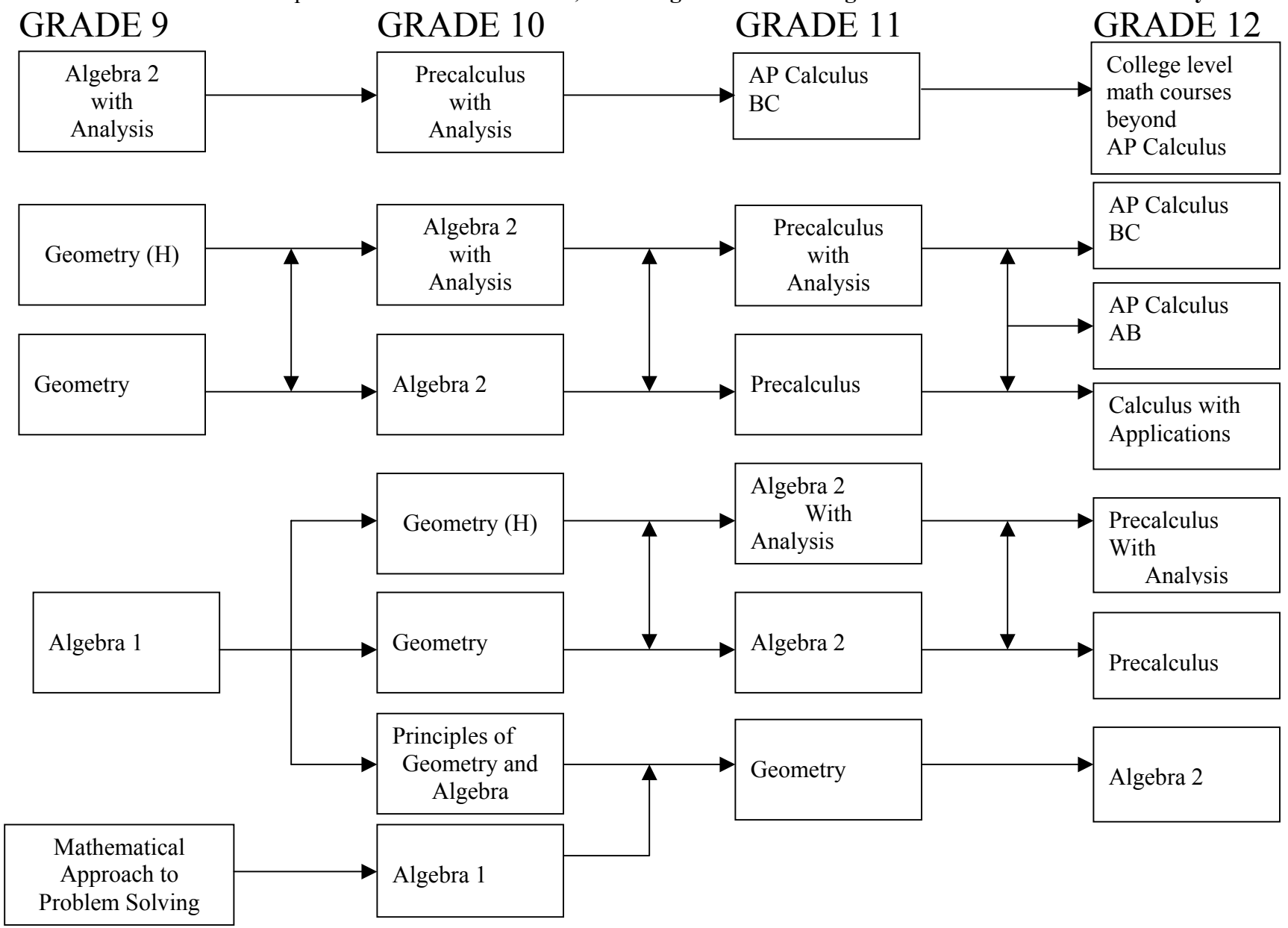
Maryland Content Standard: Knowledge of Number Relationships and Computation
Students will describe, represent, and apply numbers and their relationships, and they will estimate and compute using mental strategies, paper/pencil, and technology.

Expectation 4.2
The student will estimate and compute using mental strategies, paper and pencil, and technology.

Indicators for Algebra 2 and ***Algebra 2 with Analysis***		Indicators for Precalculus and ***Precalculus with Analysis***	
4.2.A2.1	perform operations on complex numbers.	***4.2.PC.1***	***determine the product or quotient of two complex numbers in polar form.***
4.2.A2.2	perform operations on matrices.	***4.2.PC.2***	***determine a power or the roots of a complex number using DeMoivre's Theorem.***
4.2.A2.3	evaluate logarithmic expressions.	4.2.PC.3	determine the sum, difference, scalar product, and dot product of vectors in 2-space.
4.2.A2.4	evaluate expressions involving radicals and exponents.	***4.2.PC.4***	***determine the sum, difference, scalar product, dot product, and cross product of vectors in 3-space.***
		4.2.PC.5	evaluate a logarithm using the change of base rule.
		4.2.PC.6	expand powers of binomials applying the binomial theorem, factorials, and combinatorics.

High School Mathematics Sequences

Mathematics Graduation Requirement: **Four credits, including one credit in Algebra 1 and one credit in Geometry.**



Additional Mathematics Electives

Related Mathematics	(2 semesters)	Statistics and Mathematical Modeling	(2 semesters)
Consumer Mathematics	(2 semesters)	AP Statistics	(2 semesters)

Mathematics Glossary of Terms

absolute value function: a piece-wise function $f(x) = |x|$, where $f(x) \geq 0$ for all values of x

absolute value: number's distance from zero on a number line (e.g., the absolute value of 2 and the absolute value of -2 are both 2, i.e., |2| = 2 and |-2| = 2)

additive inverse: two numbers are additive inverses of each other if their sum is 0 (e.g., since -4 + 4 = 0, then -4 and 4 are additive inverses of each other)

algebraic expression: numeral and/or variable joined by any combination of the four basic operations (+, -, x, /) and involving any power(s) of numeral and/or variable (e.g., 3-8, 7x4, 4+x, y/2, n-2, $3(4+8)-7$, $y^2 -2$)

angle measure: the measure in degrees or radians of the radial distance between two rays that meet at a point

area: the size of a region measured in number of square units

arithmetic sequence: a sequence with a constant difference between consecutive terms (e.g., 2, 5, 8, 11,... is an arithmetic sequence with a constant difference of 3)

associative property: a property of addition or multiplication in which the regrouping of the addends or factors does not change the outcome of the operations [i.e., (a + b) + c = a + (b + c) and (ab)c = a(bc)]

attribute: a characteristic of an object, such as color, shape, or size

bar graph: a graphical display representing data in different categories or groups. The length of a rectangle or bar is used to represent the numerical amount

binomial: a polynomial which is the sum of two terms

box and whisker plot: a graphical display that shows the median, quartiles, and extremes of a set of data, the spread of the data and the concentration of the data. the display does not show any other specific data values

capacity: the maximum amount that can be contained by an object. Often refers to measurement of liquid

cardinal numbers: the counting numbers (1, 2, 3...)

centroid: the point of intersection of the medians of a triangle

circle graph: a graphical display that shows data as parts of a whole circle

circle: the locus of all points in a plane at a given distance, the radius, from a fixed point in the plane, the center

circular function: a function that is defined in terms of a unit circle, whose center is at the origin and whose radius is one

circumcenter: the point of intersection of the perpendicular bisectors of a triangle

circumference: the distance around a circle; the formula for circumference of a circle is pi times the diameter ($c = \pi d$)

Mathematics Glossary of Terms

closed figure: the boundary of a simple 2-dimensional region, including shapes with straight and curved sides

collinear points: a set of points lying on the same line

combinations: a set of items selected, without regard to order, from a given set of items

commutative: property a property of addition or multiplication in which the sum or product stays the same when the order of the addends or factors is changed (i.e., a + b = b + a and ab = ba)

complex number: any number that can be written in the form $a+bi$, where *a* and *b* are real numbers and *i* is the imaginary unit

composition of functions: the composition of the functions $f(x)$ and $g(x)$ is $f(g(x))$

concrete: physical objects used to represent mathematical situations

concurrent lines: lines that have a common point

congruency: geometric figures having the same size and shape; all corresponding parts of congruent figures have the same measure

conic (conic section): a curve determined by the intersection of a plane with a double right cone

continuous function: a function is continuous at the point (x_1, y_1) if it is defined at that point and passes through that point without a break

coordinate plane: a 2-dimensional system in which the coordinates of a point are its distances from two intersecting perpendicular lines called axes. The formal name for this system is Cartesian coordinate system

counting technique: methods to determine the number of possible outcomes of an event. Some of the methods are tree diagram, list, rules for multiplication, combinations, and permutations

curve fitting: the sketching of a line or curve to best describe a relationship between two variables on a scatter plot

deductive reasoning: a series of logical steps in which a conclusion is drawn directly from a set of statements that are known or assumed to be true. (e.g., if 5 + 4 = 9 and 6 + 3 = 9 , then 5 + 4 = 6 + 3)

diagonal: for a polygon in the plane, any line segment joining non-adjacent vertices. For a polyhedron in space, a line segment joining two vertices not in the same face

dilation: a transformation which produces a figure similar to the original by proportionally shrinking or stretching the figure

dimensional analysis: a method of converting units within a measurement system

direct variation: a relationship in which the ratio of two variables is constant

discrete mathematics: the study of mathematical properties of sets and systems that have only a finite number of elements.

Mathematics Glossary of Terms

distributive property: a property which establishes a relationship between multiplication and addition such that multiplication distributes across the addition [i.e., a(b+c) = ab + ac]

divisibility (rules of): special tests to determine if a particular integer is a factor of a given number, (e.g., a number is divisible by 10 if it ends in a 0)

domain: the set of input values for a function

elapsed time: the amount of time between a beginning time and an ending time

ellipse: the locus of all points in a plane such that the sum of the distances from two given points in the plane, the foci, is constant

equally likely outcomes: events in a sample space that have the same probability of occurring

equation: a mathematical sentence of equality between two expressions (e.g., n + 50 = 75 or 75 = n + 50 means that n + 50 must have the same value as 75)

equivalent: numbers or expressions that have the same value

estimation: the process of finding a number close to an exact amount

euclidean geometry: the geometry (plane and solid) based on Euclid's postulates

event: one of the many occurrences that can take place during a probability activity

expanded notation: the sum of terms representing a quantity

experimental probability: a probability calculated from the results of an experiment

explicit relationship: a sequence rule using the number of the term to define the function [e.g., in the sequence 3, 6, 9,..., the explicit rule is f(n) = 3n where n is the number of the term and f(n) is the value of the term]

exponent: a number which is placed to the right of and above another number (base). the value of the exponent determines how many times the base is used as a factor (e.g., $3^4 = 3 \times 3 \times 3 \times 3$; {3 is the base and is used as a factor 4 times} the exponent is 4)

exponential function: a function whose general equation is a $y=ab^x$ or $y=ab^{kx}$, where a, b, and k stand for constants.

expression: a mathematical phrase with no equal sign such as 6, 3n, or + 4

face: a plane surface of a three-dimensional figure

factorial: the value of the factorial of a whole number n (written $n!$) is the product of all the natural numbers from 1 to n (e.g. $4! = 1 \cdot 2 \cdot 3 \cdot 4$)

factors: the numbers or terms multiplied in an expression

formula: an equation that states a fact or rule (e.g., a=*l* w)

frequency table: a display to show how often items, numbers, or a range of numbers occur

Mathematics Glossary of Terms

function notation: a notation that describes a function. for a function f, when x is a member of the domain, the symbol $f(x)$ denotes the corresponding member of the range [e.g., an equation of a function might be $f(x) = x+3$]

function: a relationship in which every value of x has a unique value of y (e.g., the relation $y = 2x + 1$ is a function because for every different x, there is one and only one y)

geometric mean: the terms between any two nonconsecutive terms of a geometric sequence

geometric sequence: a sequence with a constant ratio between two consecutive terms. Each successive term is found by multiplying the preceding term by the preceding term by the ratio. (e.g., 1, 2, 4, 8, 16, …is a geometric sequence with a ratio of 2.)

graph: a pictorial representation of information or relationships between numbers

histogram: a graphical display representing continuous data in different categories or groups

horizontal asymptote: the line $y = b$ is a horizontal asymptote for a function $f(x)$ if $f(x) \to b$ as $x \to \infty$ or as $x \to -\infty$

hyperbola: the locus of all points in a plane such that the absolute value of the difference of the distances from two given points in the plane, the foci, is constant

identity: a statement of equality between two expressions which is true for all values of the variable for which the expressions are defined (e.g. $\sin^2 x = 1 - \cos^2 x$)

imaginary number: a complex number of the form $a = bi$ where $b \neq 0$ and i is the imaginary unit

incenter: the point of intersection of the angle bisectors of a triangle

indirect measurement: a measurement which is found by using a formula or other strategy and not actually measuring something (e.g., finding the height of a tree without actually holding a ruler next to it)

inductive reasoning: a type of type of mathematical reasoning which involves observing patterns and using those observations to make generalizations

inequality: a mathematical sentence in which the value of the expressions on either side of the relation symbol are unequal. relation symbols include > (greater than), < (less than), ≥greater than or equal to), < (less than or equal to), (e.g., $x < y$, $7 > 3$, $n = 4$).

inference: a conclusion drawn from data

integer: a set of whole numbers and its opposites (i.e. …..-3, -2, -1, 0, 1, 2, 3, ….)

inverse function: two functions are inverse functions if and only if $f(g(x)) = g(f(x)) = x$ for all values of x

inverse operations: two operations that "undo" each other (e.g., addition and subtraction)

inverse variation: a relationship in which the product of two variables is constant

Mathematics Glossary of Terms

irrational numbers: numbers which cannot be represented by an exact ratio of two integers. (e.g., the square root of 2)

limit: the value that a function or sequence approaches as its domain values approach some number or increases/ decreases without bound

line graph: a graphical representation using points connected by line segments to show how something changes over time

line of best fit: a line drawn on a scatter plot to estimate the relationship between two sets of data

line plot: a graph using marks (e.g., x, •) above a number on a number line to show the frequency of data

linear function: a function with no exponents other than one and with no products of the variables (e.g., y=x+4, y= -4, and 3x-4y = 1/2 are linear functions); in a rectangular coordinate system, the graph of a linear function is a line

locus: a set of points and only those points that satisfy a given set of conditions

logarithmic function: $y = log_a x,\ a > 0$ and $a \neq 1$, which is the inverse of the exponential function $y = a^x$

manipulatives: tools, models, blocks, tiles, and other objects which are used to explore mathematical ideas and solve mathematical problems

matrices: rectangular arrays of numbers arranged in rows and columns

maxima: the greatest value in a data set or the greatest value of a function

mean: in a collection of data, the sum of all the data divided by the number of data

measures of central tendency: numbers which tend to cluster around the "middle" of a set of values. three such numbers are mean, median, and mode

median: the middle number (or the mean of the two middle numbers when necessary) in a collection of numbers that is arranged in order from least to greatest

minima: the least value in a data set or the least value of a function

mode: the number(s) that occur(s) most often in a collection of data

model: to represent or show mathematical ideas and relationships and real-world situations using objects, pictures, graphs, tables, functions, and other methods

monomial: a polynomial with only one term

multiple: the product of a whole number and any other whole number

multiplicative inverse: two numbers are multiplicative inverses of each other if their product is 1 (e.g., since 4 x 1/4 = 1, 1/4 and 4 are multiplicative inverses)

Mathematics Glossary of Terms

oblique (slant) asymptote: the oblique line *l* is a slant asymptote for a function $f(x)$ if the graph of $f(x)$ approaches *l* as $x \to \infty$ or as $x \to -\infty$

one-dimensional: a figure that has length but no width or height

ordinal numbers: numbers used to express order (e.g., 1st, 2nd, 3rd)

orthocenter: the point of intersection of the altitudes of a triangle

outcome of an event: one of the possible occurrances in a probability situation

parabola: the locus of all points in a plane that are the same distance from a given point, the focus, and a given line, the directrix

parallel(ism): lines that lie in the same plane and never meet. also, planes lying in space that never meet

parametric functions and relations: used to describe a graph in which multiple dependent variables defined in terms of an independent variable (e.g. $x = 3t, y = t - 2$)

patterns: regularities in situations such as those in nature, events, shapes, designs, and sets of numbers (e.g., spirals on pineapples, geometric designs in quilts, the number sequence 3, 6, 9, 12, . . .)

percent: a special ratio that compares a number to 100 and uses the % sign (e.g., 1/2 = 50% and 2/3 = 66 2/3%)

perimeter: the distance around a geometric shape

periodic function: a function with the property that there exists a number $p > 0$ (called the period) such that $f(x) = f(x + p)$ for all values of x

permutation: a set of items selected, with regard to order, from a given set of data

perpendicular(ity): lines in the same plane which intersect to form a right angle

pictograph: a graphical representation that shows numerical information by using picture symbols

piece-wise function: a function in which different equations are used for different intervals of the domain

place value: the value of a digit as determined by its position in a number (e.g., in the number "11" the one is worth either 10 or 1, depending on the position)

polar: a coordinate system that determines the location of a point by its distance from a point (the pole), and the angle of rotation relative to a horizontal ray from the pole (the polar axis)

polynomial degree: the highest power or sum of powers in any term of a given polynomial

polynomial term: an algebraic expression that is an addend in a polynomial expression

polynomial: a mathematical expression consisting of a sum of terms each of which is a product of a constant and one or more variables

Mathematics Glossary of Terms

population: a group of people, objects, or events that fit a particular description

power function: a function in the form $y = b^x$, where b is a real number

power: a number expressed using an exponent (e.g., the number 5^3 is read five to the third power or five cubed)

precision: the smallest place value to which an approximate number or measurement is expressed (e.g., if pi is represented as 3.14, then its precision is .01)

prism: a three-dimensional figure with parallelogram faces and two parallel, congruent bases

probability of an event: a number that represents the likelihood that the event will occur

properties of operations: mathematical principals that are always true (e.g., commutative, associative, distributive, inverses)

proportion: an equation of the form a/b=c/d which states that the two ratios are equivalent

pythagorean theorem: the sum of the squares of the lengths of the two legs (a, b) of a right triangle is equal to the square of the length of the hypotenuse (c). The formula is $a^2 + b^2 = c^2$

quadratic function: a function of the second degree [i.e., a function of the form $f(x) = ax^2 + bx + c$]; in a rectangular coordinate system, the graph of a quadratic function is a parabola

radical function: a function that contains a radical expression with the variable in the radicand

radical: another name for the roots of numbers, such as the square root of 5 or $\sqrt{5}$

range (of data set): the difference between the greatest and the smallest numbers in a set of data (e.g., the range of 2, 7, 13, and 17 is 15)

range: the set of output values for a function

rate: a ratio comparing two different units (e.g., miles per hour)

ratio: a comparison of two whole numbers by division

rational function: the quotient of two polynomials in the form $f(x) = \frac{g(x)}{h(x)}$, where $h(x) \neq 0$

rational number: a real number that can be written as a quotient/ratio of two integers a/b, where b does not equal 0; a repeating or terminating decimal, integer, fraction, or whole number

real number: any number that is either rational or irrational

recursive relationship: a function rule which uses the value of the preceding term in the definition

reflection (flip): a transformation which produces the mirror image of a figure (i.e., flipping a figure across a line)

root: a solution of the equation $P(x) = 0$

Mathematics Glossary of Terms

rotation (turn): a transformation obtained by rotating a figure around a fixed point (i.e., turning a figure about a point)

sample space: the set or collection of all possible outcomes of a probability experiment

scale drawing: a scaled representation of physical objects or drawings

scale: choice of increments and range of numbers on an axis

scatter plot: a graphical representation consisting of ordered pairs possibly showing a relationship between two variable quantities

scientific notation: representation of a number as the product of a number between 1 and 10 and a power of 10; used especially for very small or very large numbers (e.g., 6,900,000 = 6.9×10^{6} or .00069 = 6.9×10^{-4})

sequence: a function whose domain is the natural numbers

series: an indicated sum of terms of a sequence

similarity: two or more figures having the same shape but not necessarily the same size

simulation: a representation of a situation or problem with a similar but simpler model or a more easily manipulated model in order to determine experimental results

slides: a transformation in which the figures moves in a linear direction

slope: the ratio of the rise to the run of any two distinct points on a line

solutions: the values of a variable that make an equation or inequality a true statement

statistical investigation: a procedure for obtaining data and drawing conclusions or making decisions on the basis of available data

stem and leaf plot: a method of organizing data for the purpose of comparison where the "leaf" is the number in the smallest place value and the "stem" includes the numbers in the larger place values

surface area: the sum of the areas of the faces of a solid figure

symmetry: a figure has symmetry if there exists some line or point through which all points of the figure can be reflected to generate another point on the figure

theoretical probability: a probability of a given event calculated from mathematical counting techniques. (e.g., the chance of getting heads when flipping a coin is 1 out of 2.)

three-dimensional: an object that has length, width, and height

transformation: a rule for moving every point in a plane figure to a new location.

translation (slide): a transformation that slides a figure a given distance in a given direction

trend: the general drift, tendency, or direction of a set of data

Mathematics Glossary of Terms

trigonometric function: a function whose domain is the set of all angle measures, and whose range is one of the six trigonometric ratios of the angle measure

trigonometric ratio: a comparison of the measures of the lengths of two sides of a right triangle expressed in fractional or decimal form; there are six trigonometric ratios (sine, cosine, tangent, cotangent, secant, and cosecant) associated with any angle

trigonometry: the study of right triangle measurements and ratios, useful for calculating indirect measurements

two-dimensional: a figure that has length and width but not height (i.e., a plane figure such as a rectangle or circle)

valid argument: an explicit demonstration or proof that has been shown to be true

validate: to give evidence that a solution or process is correct

variability: numbers that describe how spread out a set of data is. (e.g., range and quartile)

variable: a letter or symbol which represents one or more numbers

vector: a quantity with both magnitude and direction

vertex (of a conic section): a point at which a conic section intersects its axis of symmetry

vertex (vertices): the points where two line segments come together (corners)

vertical asymptote: the line $x = a$ is a vertical asymptote for a function $f(x)$ if $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$ from either the left or the right

volume: the amount of space enclosed in a three-dimensional figure, measured in cubic units

whole numbers: the numbers in the set {0, 1, 2, 3, …}

zero: a value of x for which $f(x) = 0$