Mathematics Curriculum Framework
Montgomery County Public Schools
July 2001

Goal:

The goal of the Montgomery County Public Schools pre-K-12 mathematics program is for all students to achieve mathematical proficiency by developing both conceptual understanding and procedural fluency. The end result is the ability to think and reason mathematically and use mathematics to solve problems in authentic contexts.

Overarching Enduring Understandings:

- Mathematics is the study of patterns and relationships.
- Mathematics is a language consisting of carefully defined terms and symbols.
- Mathematics is a tool used to solve problems in everyday life.
- Technology influences the mathematics that is taught and essential for our world.

The Content of Mathematics:

Mathematics is a tool we use to understand and interpret our world. In our increasingly technological economy, those who can understand and apply mathematics have significantly enhanced opportunities to achieve success in continuing education and in life. The key to opening the door to these opportunities is a deep understanding of important mathematical concepts and procedures.

The integration of both mathematical concepts and processes is essential for meaningful understanding of mathematics. In this framework, the concepts of mathematics are organized under six strands: Algebra/Patterns/Functions, Geometry, Measurement, Statistics, Probability, and Number Relationships and Computation. These concepts are developed through the four mathematical processes that are organized into four strands: Problem Solving, Communication (including representation), Reasoning, and Connections.

The mathematical content must be coherent and vertically articulated across the grades.

- In the elementary school years, students develop proficiency with number concepts and operations. For this to occur, students' experiences with the concept of number must be connected to mathematical concepts in geometry, algebraic reasoning, and data analysis. At the same time, proficiency with mathematical facts and skills must be developed so that students are facile in their application of mathematics to solve problems.
- In the middle grades, students extend their mathematical proficiency through their work with rational numbers, proportional reasoning, measurement, and data analysis. They continue the development of a deep understanding of important algebraic and geometric concepts as well as mathematical ways of thinking. The expectation is for all students to be successful in the formal study of algebra and other academically challenging mathematics courses. The PreK-7 indicators and the checked grade 8 indicators identify the prerequisite knowledge and understanding necessary for success in Algebra I, Geometry, and beyond.
- In high school, all students pursue rigorous mathematics coursework that includes concepts such as functions, statistics, calculus, and discrete mathematics. Their understanding of mathematics enables students to see the connections among algebra, geometry, statistics, and discrete mathematics. Students must be able to visualize, represent, and analyze situations within the discipline as well as in other areas using mathematical terms.

Instructional Approach:
Learning with understanding is essential for developing mathematical proficiency. According to the National Research Council's report *Adding It Up*, mathematical proficiency implies expertise in handling mathematical ideas. Students with mathematical proficiency understand basic concepts, are fluent in performing basic operations, reason clearly, formulate, represent, and solve mathematical problems, and maintain a positive outlook toward mathematics. (Kilpatrick, 2001) These components of mathematical proficiency are interwoven and interdependent. Instruction must help students develop increasingly efficient strategies for producing basic facts or single digit number combinations rapidly and accurately. This development leads to proficiency with basic facts. Students' proficiency with multi-digit numerical operations develops through understanding and reasoning, as well as meaningful practice. Students' understanding of operations serves as a foundation for reasoning about mathematics. The interdependence and connections among all six mathematical strands fosters the development of mathematical proficiency. Students must be actively engaged in learning experiences that are designed to deepen, connect, and build on students' knowledge. Communication is an essential part of mathematics education. Instruction must provide students with opportunities for speaking, reading, writing, representing, and listening in mathematics classrooms so that they will learn to communicate mathematically. (Principles and Standards for School Mathematics, 2000) Technology is a tool for investigation and problem solving that enhances learning of mathematics. The use of technology should support the development of mathematical proficiency.

Mathematics teaching and learning must be challenging and rigorous with an emphasis on problem solving and reasoning. The curriculum makes a distinction between problem solving as a general process and the solution of specific word problems that demonstrate application of mathematical skills. A mathematical problem is something that you do not already know how to do. Problem solving is the process of transforming something that you do not know how to do into something familiar. (Steen, 1997) The mathematical problem solving situations that students encounter should include problems that require broader thinking than traditional word problems demand. (Burns, 1992) Word problems are a means for practicing computation. For example, a traditional word problem might ask: *How much change would you receive from a $10 bill if you spend $2.75?* The intent of this problem is to practice subtraction. Problem solving, on the other hand, should require students to develop a plan, execute the plan, and establish a purpose for learning to compute. For example, the previously cited word problem becomes a problem solving situation when it is restated as follows. *Your change from a $10 bill when you spend $2.75 is $7.25. The digits in your change are the same as the digits in what you spent. What other amounts could you spend so that your change has the same digits?* (Burns, 1992) Problem solving must occur at every grade level and be the primary focus in mathematics instruction.

Differentiated instruction addresses student strengths, interests, and learning styles and should be paced to make the curriculum accessible to everyone. Flexible and varied grouping practices enhance the opportunity to receive expanded, intensive, enriched, and accelerated curriculum at all instructional levels as warranted by students' needs. A balance needs to be achieved so that all students have the opportunity to work in homogenous and heterogeneous groups. The curriculum is designed so that all students have the necessary skills and understanding for success in Algebra I, Geometry, and beyond. The pathway for students to take Algebra I in eighth grade is clearly laid out in the PreK-7 indicators and the checked eighth grade indicators. Differentiation and grouping practices must be implemented to ensure that all students have the opportunity to be successful in Algebra I in eighth grade. The Grade 8 curriculum is designed to meet the needs of the smaller number of students who require additional support for success in Algebra I, Geometry, and other rigorous mathematics courses in high school.

Assessment is an ongoing process that guides instruction and monitors student progress to include mastery of mathematics content and higher level thinking skills. Pre-assessment, formative, and summative assessments provide for student, peer, and teacher evaluation. These types of assessment enable teachers to modify their instruction to support improved learning at each grade level for all students. Assessment should be focused on the development and achievement of mathematical proficiency. (Kilpatrick, 2001)

Documents and Concepts Considered in this framework:

Advanced Placement. The College Board/Educational Testing Service. http://www.ets.org/satets.html

Beaton, Albert E., Ina V.S. Mullis, Michael O. Martin, Eugenio J. Gonzalez, Dana L. Kelly, and Teresa A. Smith. Mathematics Achievement in the Middle School Years: IEA's Third International Mathematics and Science Study (TIMSS). Chestnut Hill, MA: Center for the Study of Testing, Evaluation, and Educational Policy, Boston College, 1996. <http://www.timss.org>

Benchmarks for Science Literacy; Project 2061, American Association for the Advancement of Science. New York: Oxford UP, 1993.

Burns, Marilyn. About Teaching Mathematics, A K-8 Resource. Math Solutions Publications, 1992

California Math Standards. < http://www.step.k12.ca.us/resources/MathStandards.html>.

Charles, Randall, and Joanne Lobato. Future Basics: Developing Numerical Power. Golden, CO: National Council of Supervisors of Mathematics, 1998.

First in the World Consortium Mathematics and Science Standards. Naperville, IL: North Central Regional Educational Laboratory, 2001. <http://www.ncrel.org/re/fitwsp/.>

High School Core Learning Goals: Mathematics; Maryland School Performance Program. Baltimore, MD: Maryland State Department of Education, 1999. 5 June 2001 <http://www.mdk12.org/mspp/high_school/what_will/mathematics/index.html> or <http://www.mdk12.org/mspp/high_school/what_will/mathematics/mathematics_goals99.pdf>

Keys to Math Success A Report from the Maryland Mathematics Commission. Baltimore, MD: Maryland State Department of Education, June 2001.

Kilpatrick, Jeremy, Jane Swafford, and Bradford Findell, eds. Adding It Up: Helping Children Learn Mathematics. Washington, DC: National Academy Press, 2001.

Mullis, Ina V.S. et al. Mathematics Benchmarking Report TIMSS 1999 – Eighth Grade. Boston, MA: International Study Center, Boston College, International Association for the Evaluation of Educational Achievement. April 2001 <http://www.timss.org>

Maryland Mathematics Content Standards. Baltimore, MD: Maryland State Department of Education, 2000. 5 June 2001 <http://www.mdk12.org/practices/support_success/mspap/mathematics/content_standards.html> or <http://www.mdk12.org/share/standards/constds_math.pdf>

Maryland Learner Outcomes: Maryland School Performance Assessment Program; Mathematics. Baltimore, MD: Maryland State Department of Education, 2000. 5 June 2001 <http://www.mdk12.org/practices/support_success/mspap/mathematics/learner_outcomes.html> or <http://www.mdk12.org/share/mlo/mlo_math.pdf>

Marzano, Robert J., and Debra J. Pickering. Dimensions of Learning. 2nd ed. Alexandria, VA: Association for Supervision and Curriculum Development; Aurora, CO: Mid-continent Regional Educational Laboratory, 1997.

Mullis, Ina V.S., Michael O. Martin, Albert E. Beaton, Eugenio J. Gonzalez, Dana L. Kelly, and Teresa A Smith. Mathematics Achievement in the Primary School Years: IEA's Third International Mathematics and Science Study (TIMSS). Chestnut Hill, MA: Center for the Study of Testing, Evaluation, and Educational Policy, Boston College, 1997.

Primary Mathematics Syllabus. Singapore: Ministry of Education, Curriculum Planning and Development Division, 2000.

Principles and Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics, 2000.

Scholastic Achievement Test. The College Board/Educational Testing Service. <http://www.ets.org/satets.html>.

Science for All Americans: A Project 2061 Report on Literacy Goals in Science, Mathematics, and Technology. Washington, DC: American Association for the Advancement of Science, 1989

Steen, Lynn Arthur. Why Numbers Count Quantitative Literacy for Tomorrow's America. New York: College Entrance Examination Board, 1997

Tomlinson, Carol Ann. The Differentiated Classroom: Responding to the Needs of All Learners. Alexandria, VA: Association for Supervision and Curriculum Development, 1999.

Wiggins, Grant, and Jay McTighe. Understanding by Design. Alexandria, VA: Association for Supervision and Curriculum Development, 1998.