Two-Year Algebra 2 A
Semester Exam Review
2015–2016
Exam Formulas

General Exponential Equation: \( y = ab^x \)

Exponential Growth: \( A(t) = A_0 (1+r)^t \)

Exponential Decay: \( A(t) = A_0 (1-r)^t \)

Continuous Growth: \( A(t) = A_0 e^{rt} \)

Continuous Decay: \( A(t) = A_0 e^{-rt} \)

Compound Interest (\(n\) compoundings per year): \( F(t) = P \left( 1 + \frac{r}{n} \right)^{nt} \)

Compound Interest (continuous compounding): \( F(t) = Pe^{rt} \)

\( \log_b N = p \) if and only if \( b^p = N \)

The average rate of change for a function \( f \) on the interval \([a,b]\): \( \frac{f(b) - f(a)}{b - a} \)
Unit 1, Topic 1

1. Let $f$ and $g$ be functions that are inverses of each other.

   Complete the following statements.
   
   a. If the point $(a,b)$ is on the graph of $f$, then the point ________ is on the graph of $g$.
   
   b. If $f(3) = 7$, then $g(7) =$ _____.
   
   c. The graphs of $f$ and $g$ are symmetric with respect to the line ________________.
   
   d. The range of $f$ is the same as the ______________ of $g$.
   
   e. The domain of $f$ is the same as the ______________ of $g$.

2. Let $f$ and $g$ be functions that are inverses of each other.

   a. Give a numerical example showing why if $f(x) = x^2$, then $g(x) \neq \frac{1}{x^2}$.

   b. Give a numerical example showing why if $f(x) = 3x$, then $g(x) \neq -3x$.

3. Let $f$ and $g$ be functions that are inverses of each other.

   a. If $f(x) = 3x - 2$, then $g(x) =$ ________________.

   b. If $f(x) = 2x + 9$, then $g(x) =$ ________________.

   c. If $f(x) = x^3 + 7$, then $g(x) =$ ________________.
4. For each graph below, sketch the inverse function on the graph to its right.

a.

b.
5. Jill sells lemonade. The profit, \( p \), in dollars is a function of the number of glasses of lemonade, \( g \), that she sells. The function that represents this relationship is 
\[ p(g) = 2g - 18. \]

a. Write a function that will represent the number of glasses that she will need to sell to earn a profit of \( p \) dollars.

b. If Jill made a profit of $32, how many glasses did she sell?

6. On a national test, a student receives a score based on the number of correct items. The score, \( s \), in points is a function of the number of correct items \( c \). The function that represents this relationship is 
\[ s(c) = 200 + 2.5c. \]

a. Write a function that will give the number of correct items that it will take to receive a score of \( s \).

b. A student received a score of 325. How many items did the student get correct?
Unit 1, Topic 2

7. Write the following expression as a radical.
   a. \( \frac{1}{5} \) \( x^5 \)
   b. \( \frac{1}{3} \) \( y^\frac{1}{3} \)
   c. \( \frac{2}{3} \) \( z^3 \)

8. Determine the exponent that goes into the box.
   a. \( \frac{1}{\sqrt{x}} = x \)
   b. \( 5\sqrt{x^4} = x \)
   c. \( \left( \frac{x^3}{x^5} \right)^{18} = x \)
   d. \( (\sqrt{x})^6 = x \)
   e. \( \frac{x}{x^6} = x^{\frac{2}{3}} \)

   a. \( 81^{\frac{1}{3}} \)
   b. \( 25^{\frac{1}{2}} \)
   c. \( 8^{\frac{2}{3}} \)

10. Sally solves the radical equation \( 3\sqrt{x} = -15 \) and obtains the solution \( x = 25 \). Is this solution extraneous? Justify your answer.

11. Giacomo solves the radical equation \( \sqrt{3x+4} = x \) and obtains the solutions \( x = -1 \) and \( x = 4 \). Determine if either of the solutions are extraneous.

12. Johnny solves the radical equation \( \sqrt[3]{x} = -2 \) and obtains the solution \( x = -8 \). Is this solution extraneous? Justify your answer.
13. Sketch the equations below.

a. \( y = -\sqrt{x} \)

b. \( y = \sqrt{x} \)

c. \( y = \sqrt{x + 2} - 3 \)

d. \( y = -\sqrt{x} + 2 \)

14. Let \( f(x) = \sqrt{x} + 7 \).

a. What is the domain of \( f \)? _________________________________

b. What is the range of \( f \)? _________________________________

c. On what interval is the function decreasing? ______________________
15. Solve the equation \( \sqrt[3]{5x} = 10 \)

16. A bike rider sees a deer crossing the street and puts on the brakes. The distance that he travels, in feet, can be modeled by the function \( D(s) = 30 + 5\sqrt{s} \), where \( s \) is the speed of the bike in miles per hour. If the bike travels a distance of 58 feet, what was his speed?

17. The population, \( P \), of a town can be modeled by the function \( P(x) = 60,000\sqrt[3]{x-1970} \), where \( x \) is the year. In what year was the population 120,000?
Unit 1, Topic 3

18. There are four functions represented by tables below. They are either exponential or logarithmic. For each table, write a function equation.

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>f(x)</th>
<th></th>
<th>x</th>
<th>g(x)</th>
<th></th>
<th>x</th>
<th>h(x)</th>
<th></th>
<th>x</th>
<th>m(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
<td></td>
<td>b</td>
<td></td>
<td></td>
<td>c</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3</td>
<td></td>
<td>5</td>
<td>1</td>
<td></td>
<td>-3</td>
<td>1/8</td>
<td></td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1/3</td>
<td></td>
<td>25</td>
<td>2</td>
<td></td>
<td>-2</td>
<td>1/4</td>
<td></td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1/9</td>
<td></td>
<td>125</td>
<td>3</td>
<td></td>
<td>-1</td>
<td>1/2</td>
<td></td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1/27</td>
<td></td>
<td>625</td>
<td>4</td>
<td></td>
<td>0</td>
<td>1</td>
<td></td>
<td>1,000</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1/81</td>
<td></td>
<td>3125</td>
<td>5</td>
<td></td>
<td>1</td>
<td>2</td>
<td></td>
<td>10,000</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1/243</td>
<td></td>
<td>15625</td>
<td>6</td>
<td></td>
<td>2</td>
<td>4</td>
<td></td>
<td>100,000</td>
<td>5</td>
</tr>
</tbody>
</table>

19. For each equation in column 1, choose the interval in which the solution lies from column 2.

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \log_2 18 = x )</td>
<td>( x ) is between 0 and 1</td>
</tr>
<tr>
<td>b. ( 3^x = 10 )</td>
<td>( x ) is between 1 and 2</td>
</tr>
<tr>
<td>c. ( \log 20 )</td>
<td>( x ) is between 2 and 3</td>
</tr>
<tr>
<td>d. ( 5^{-x} = \frac{1}{6} )</td>
<td>( x ) is between 4 and 5</td>
</tr>
</tbody>
</table>

20. Write the logarithmic equation that is equivalent to each exponential equation

a. \( 4^2 = 16 \)   b. \( 10^3 = 1000 \)   c. \( e^0 = 1 \)

21. Write the exponential equation that is equivalent to each logarithmic equation.

a. \( \log_{10} \frac{1}{100} = -2 \)   b. \( \ln \frac{1}{e} = -1 \)   c. \( \log_5 81 = 2 \)
22. Does each function below represent exponential growth or decay?
   
   a. \( f(x) = \left(\frac{1}{2}\right)^x \)  
   b. \( g(x) = 3^{-x} \)  
   c. \( h(x) = 5^x \)  

23. Write an exponential function in terms of time \( t \) (\( t \) in years) for each situation.
   
   a. There are 300 bacteria at time 0. The bacteria has a continuous growth rate of 70% per year.
   
   b. The population of a town is currently 2000. The population is growing at an annual rate of 11% per year.
   
   c. Jack puts $500 into a savings account. It earns interest at a nominal annual rate of 6% per year, compounded monthly.
   
   d. The number of deer in a forest is decreasing at an annual rate of 8%. There are currently 700 deer in the forest.
   
   e. The number of gnats in a swamp decreases at a continuous decay rate of 12% per year. There are currently 4 billion gnats in the swamp.
24. Look at the functions and their graphs below.

\[ f(x) = 2^x \quad \quad g(x) = \log_3 x \]

\[ h(x) = 10^{-x} \quad \quad p(x) = -\ln x \]

Several properties are listed below. For each property, write the function(s) that have this property. You may use \( f, g, h, \) or \( p \) as your answers.

a. _________________ The graph of the function has a horizontal asymptote of \( y = 0 \).

b. _________________ The function has a range of all real numbers.

c. _________________ The function is increasing on its entire domain.

d. _________________ The graph of the function has a \( y \)-intercept at the point \( (0,1) \).

e. _________________ The domain of the function is the positive real numbers.

f. _________________ The graph of the function has a vertical asymptote of \( x = 0 \).

g. _________________ The function has a domain of all real numbers.

h. _________________ The function is decreasing on its entire domain.

i. _________________ The graph of the function has an \( x \)-intercept at the point \( (1,0) \).

j. _________________ The range of the function is the positive real numbers.
25. Each function below is a transformation of the function \( f(x) = e^x \). After each given transformation, write the function rule.

a. The graph of function \( g \) is the graph of \( f(x) = e^x \) translated one unit to the right. 
\[ g(x) = \] 

b. The asymptote of the graph of function \( h \) has the equation \( y = -4 \).
\[ h(x) = \] 

c. The graph of function \( p \) is the graph of \( f(x) = e^x \) reflected across the \( x \)-axis.
\[ p(x) = \] 

d. The graph of function \( s \) is the graph of \( f(x) = e^x \) reflected across the \( y \)-axis.
\[ s(x) = \] 

26. Each function below is a transformation of the function \( f(x) = \log_2 x \). After each given transformation, write the function rule.

a. The graph of function \( g \) is the graph of \( f(x) = \log_2 x \) translated two units to the left.
\[ g(x) = \] 

b. The asymptote of the graph of function \( h \) has the equation \( x = 2 \).
\[ h(x) = \] 

c. The graph of function \( p \) is the graph of \( f(x) = \log_2 x \) reflected across the \( x \)-axis.
\[ p(x) = \] 

d. The graph of function \( s \) is the graph of \( f(x) = \log_2 x \) reflected across the \( y \)-axis.
\[ s(x) = \] 

27. Evaluate the following logarithms

a. \( \log_3 9 = \) 

b. \( \log_4 \frac{1}{16} = \) 

c. \( \log 10000 = \) 

d. \( \ln e = \) 

e. \( \log_6 1 = \) 

f. \( \ln \frac{1}{e} = \) 

g. \( \log_4 8 = \) 

h. \( \log_{100} 10 = \)
28. Solve each equation. Show the work that leads to your solution. Your answer must be exact or accurate to three places after the decimal point.

a. \[ 7 \cdot 10^x = 56 \]

b. \[ e^{10} = 1000 \]

29. Jack puts $2500 in the bank at a nominal rate of interest of 3% a year.

a. If the interest is compounded monthly (12 times a year), how much money will Jack have after 5 years? Round your answer to the nearest cent.

b. If the interest is compounded continuously, how much money will Jack have at the end of 5 years? Round your answer to the nearest cent.
30. A house had a value of $100,000 at the beginning of 2015. Its value increases at the rate of 30% per year.

   a. Write a function for the value $V(t)$ of the house after $t$ years.
      
      $V(t) = ______________________________$

   b. What is the value of the house at the beginning of 2019 ($t = 4$)?

   c. What is the average rate of change of the value of the house from the beginning of 2015 ($t = 0$) to the beginning of 2019 ($t = 4$). Be sure to include the units in your answer.

31. A boat has a value of $1,000,000 on January 1, 2015. Its value decreases at a rate of 20% per year.

   a. Write a function for the value $V(t)$ of the boat after $t$ years.
      
      $V(t) = ______________________________$

   b. What is the value of the boat at the beginning of 2020 ($t = 5$)?

   c. What is the average rate of change of the value of the house from the beginning of 2015 ($t = 0$) to the beginning of 2020 ($t = 5$). Be sure to include the units in your answer.